2021 年组合、图论及数论研讨会

会议手册

南方科技大学 · 数学系

中国 · 广东 · 深圳

2021 年 11 月 4 日 – 11 月 7 日

2021 年组合、图论及数论研讨会

会议宗旨: 本次会议旨在加强国内组合、图论与数论数学领域之间同行的交流和合作，及时了解相关领域的前沿研究动态，深入探讨相关课题。

主办单位: 南方科技大学理学院数学系

组织委员会:

向　青　南方科技大学

李才恒　南方科技大学

王　琦　南方科技大学

周　悦　国防科技大学

报到时间: 2021 年 11 月 4 日

报到地点: 深圳君璞酒店大堂 (深圳市南山区留仙大道 3333 号塘朗城广场 C 座)

会议时间: 2021 年 11 月 4 日 – 2021 年 11 月 7 日

会议地点: 深圳君璞酒店会议室 2

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会议日程

11 月 5 日星期五上午

地点: 深圳君璞酒店会议室 2 /腾讯会议号：884120101

主持: 向青	
9:00–9:10	王晓明系主任讲话

主持: 向青		
9:10–9:50	李雨生	Ramsey thresholds of graphs with most degrees bounded
9:55–10:20	☕ 茶歇	
10:20–11:00	王毅	Analytic aspects of combinatorial sequences
11:05–11:45	孙智伟	On new type series for powers of π
11:45–14:00	🍴 午餐	

11 月 5 日星期五下午

地点: 深圳君璞酒店会议室 2 /腾讯会议号：884120101

主持: 孙智伟		
14:00–14:30	周悦	(Online) Perfect and almost perfect linear Lee codes
14:35–15:05	张俊	(Online) Finite geometry and Reed-Solomon codes
15:10–15:40	周海燕	(Online) Romanoff's theorem for polynomials over finite fields revisited
15:40–16:05	茶歇	
16:05–16:45	洪绍方	(Online) On the number of zeros of diagonal quartic forms over finite fields
16:50–17:30	赵立璐	稠密集合上的二次方程的零点
17:30–19:00	晚餐	

11 月 6 日星期六上午

地点: 深圳君璞酒店会议室 2 /腾讯会议号：884120101

主持: 王军		
9:00–9:40	傅士硕	(Online) Proofs of six conjectures relating permanents to combinatorial sequences
9:45–10:10	茶歇	
10:10–10:50	岳勤	Exponential sums and their applications
10:55–11:25	曹喜望	(Online) Some results on uniform mixing on abelian Cayley graphs
11:30–12:00	唐元生	(Online) On uniqueness of some girth eight graphs
12:00–14:00	午餐	

11 月 6 日星期六下午

地点: 深圳君璞酒店会议室 2 /腾讯会议号：884120101

主持: 李雨生		
14:00–14:40	袁平之	Perfect polynomials over finite fields
14:45–15:15	张先得	(Online) Suitable sets of permutations
15:20–15:50	茶歇	
15:50–16:30	吴耀琨	Maximum size of k-collapsible grid point sets
17:00–19:00	晚餐	

11 月 7 日星期日上午

地点: 深圳君璞酒店会议室 2 /腾讯会议号：884120101

主持: 吴耀琨		
9:00–9:30	张晓东	(Online) Spectral Extremal Results for Linear Forests
9:35–10:05	茶歇	
10:05–10:45	祝隐峰	Primitive matrix sets and synchronizing automata
12:00–14:00	午餐	

摘要

Some results on uniform mixing on abelian Cayley graphs

曹喜望

南京航空航天大学

In the past few decades, quantum algorithms have become a popular research area of both mathematicians and engineers. Among them, uniform mixing provides a uniform probability distribution of quantum information over time which attracts a special attention. However, there are only a few known examples of graphs which admit uniform mixing. In this paper, a characterization of abelian Cayley graphs having uniform mixing is presented. Some concrete constructions of such graphs are provided. Specifically, for cubelike graphs, it is shown that the Cayley graph $Cay(\mathbb{F}_2^{2k}; S)$ has uniform mixing if the characteristic function of S is bent. Moreover, a difference-balanced property of the eigenvalues of an abelian Cayley graph having uniform mixing is established. Furthermore, it is proved that an integral abelian Cayley graph exhibits uniform mixing if and only if the underlying group is one of the groups: $\mathbb{Z}_2^d$, $\mathbb{Z}_3^d$, $\mathbb{Z}_4^d$ or $\mathbb{Z}_2^r \otimes \mathbb{Z}_4^d$ for some integers $r \geq 1, d \geq 1$. Thus the classification of integral abelian Cayley graphs having uniform mixing is completed.

Proofs of six conjectures relating permanents to combinatorial sequences

傅士硕

重庆大学

In this talk, we prove six conjectures on the evaluations of permanents, relating them to Genocchi numbers of the first and second kinds, Euler numbers, as well as binomial Euler numbers. For example, we prove that

$$\operatorname{per}\left[\left\lfloor\frac{2j-k}{n}\right\rfloor\right]_{1\leq j,k\leq n} = 2(2^{n+1}-1)B_{n+1},$$

where $\lfloor\cdot\rfloor$ is the floor function and $B_0, B_1, \ldots$ are the Bernoulli numbers. This was previously conjectured by Zhi-Wei Sun. The talk is based on joint work with Zhicong Lin and Zhi-Wei Sun.

On the number of zeros of diagonal quartic forms over finite fields

洪绍方

四川大学

Let $\mathbb{F}_q$ be the finite field of $q = p^m \equiv 1 \pmod 4$ elements with p being an odd prime and m being a positive integer. For $c, y \in \mathbb{F}_q$ with $y \in \mathbb{F}_q^*$ non-quartic, let $N_n(c)$ and $M_n(y)$ be the numbers of zeros of $x_1^4 + ... + x_n^4 = c$ and $x_1^4 + ... + x_{n-1}^4 + yx_n^4 = 0$, respectively. In 1979, Myerson used Gauss sum and exponential sum to show that the generating function $\sum_{n=1}^{\infty} N_n(0)x^n$ is a rational function in x and presented its explicit expression. In this paper, we make use of the cyclotomic theory and exponential sums to show that the generating functions $\sum_{n=1}^{\infty} N_n(c)x^n$ and $\sum_{n=1}^{\infty} M_{n+1}(y)x^n$ are rational functions in x. We also obtain the explicit expressions of these generating functions. Our result extends Myerson's theorem gotten in 1979. This is a joint work with Drs Yulu Feng, Junyong Zhao and Chaoxi Zhu.

Ramsey thresholds of graphs with most degrees bounded

李雨生

同济大学

We shall discuss Ramsey thresholds of $K_{m,n}$ and $B_m(n) = K_m + \overline{K}_n$ for large n in some forms. In this talk, we shall report the Ramsey numbers of graphs of graphs with most degrees bounded in random graphs, and the thresholds and Ramsey thresholds of $K_{m,n}$ as $n \to \infty$. Also, we propose weak Ramsey thresholds of $B_m(n)$ as $n \to \infty$.

On new type series for powers of π

孙智伟

南京大学

In this talk, we introduce two new types of series for powers of π motivated by Ramanujan-type series and Zeilberger-type series. For example, we prove that

$$\sum_{k=0}^{\infty}(198k^2-425k+210)\frac{k^3\binom{2k}{k}^3}{4096^k}=-\frac{1}{21\pi}$$

and

$$\sum_{k=0}^{\infty}\frac{198k^2-227k+47}{\binom{2k}{k}^3}=\frac{3264-4\pi^2}{63}.$$

We will introduce our general algorithm to obtain such series, and mention various conjectures in this new direction.

On uniqueness of some girth eight graphs

唐元生

扬州大学

For any field $\mathbb{F}$ and polynomials $f_2, f_3 \in \mathbb{F}[x,y]$, let $\Gamma_{\mathbb{F}}(f_2,f_3)$ denote the bipartite graph with vertex partition $P \cup L$, where P and L are two copies of $\mathbb{F}^3$, and $(p_1,p_2,p_3) \in P$ is adjacent to $[l_1,l_2,l_3] \in L$ if and only if $p_2+l_2=f_2(p_1,l_1)$ and $p_3+l_3=f_3(p_1,l_1)$. The graph $\Gamma_3(\mathbb{F})=\Gamma_{\mathbb{F}}(xy,xy^2)$ is known to be of girth eight. When $\mathbb{F}=\mathbb{F}_q$ is a finite field of odd characteristic or $\mathbb{F}=\mathbb{F}_\infty$ is an algebraically closed field of characteristic zero, the graph $\Gamma_3(\mathbb{F})$ is conjectured to be the unique one with girth at least eight among those $\Gamma_{\mathbb{F}}(f_2,f_3)$ up to isomorphism. This conjecture has been confirmed for the case that both f_2, f_3 are monomials over $\mathbb{F}_q$, and for the case that at least one of f_2, f_3 is a monomial over $\mathbb{F}_\infty$. If one of $f_2, f_3 \in \mathbb{F}_q[x,y]$ is a monomial, it has also been proved the existence of a positive integer M such that $G=\Gamma_{\mathbb{F}_{q^M}}(f_2,f_3)$ is isomorphic to $\Gamma_3(\mathbb{F}_{q^M})$ provided G has girth at least eight. In this paper, these results are shown to be valid when the restriction on the polynomials f_2, f_3 is relaxed further to that one of them is the product of two univariate polynomials. Furthermore, all of such polynomials f_2, f_3 are characterized completely.

Analytic aspects of combinatorial sequences

王毅

大连理工大学

Total positivity is an important and powerful concept that arises in various branches of mathematics. In this talk we discuss some analytic properties of combinatorial sequences from the viewpoint of total positivity.

Maximum size of k-collapsible grid point sets

吴耀琨

上海交通大学

We use linear algebra to determine the maximum size of a k-collapsible n-dimensional grid point set. As a consequence, we can determine the phylogeny numbers of generalized Hamming graphs. This is joint work with Chengyang Qian and Yanzhen Xiong.

Perfect polynomials over finite fields

袁平之

华南师范大学

In this talk, we will give a survey on perfect polynomials over finite fields. Including recent progress and some open problems. We also will talk about a little for the well-known odd perfect numbers.

Exponential sums and their applications

岳勤

南京航空航天大学

In this talk, we mainly compute a class of exponential sums. We use semi-primitive Gauss periods and rational points of elliptic curve to calculate exponential sums. On the other hand, we apply them for coding theory.

Finite geometry and Reed-Solomon codes

张俊

首都师范大学

It is well-known that MDS codes and Reed-Solomon codes are corresponding to arcs and normal rational curves in finite geometry, respectively. In this talk, we discuss the relationship between error distance problem and deep hole problem of Reed-Solomon codes and the corresponding geometry problems. This is joint work with Daqing Wan and Krishna Kaipa.

Suitable sets of permutations

张先得

中国科学技术大学

Suitable sets of permutations are closely related to poset dimension and boxicity. In this talk, I will review their relations and present our progress on Suitable sets of permutations.

Spectral Extremal Results for Linear Forests

张晓东

上海交通大学

For a given graph F, let $ex(n, F)$ and $\lambda_{ex}(n, F)$ be the maximum edge and the spectral radius of a graph of order n without containing F as a subgraph, respectively. Understanding $ex(n, F)$ and $\lambda_{ex}(n, F)$ for various graph F is a cornerstone of extremal graph theory. In this talk, we introduce some new results on $ex(n, F)$ and $ex_{\lambda}(n, F)$ for linear forest (i.e. disjoint paths). Moreover, some open problems are included. (This talk is based on the work with Ming-Zhu Chen, A-Ming Liu, Long-Tu Yuan)

稠密集合上的二次方程的零点

赵立璐

山东大学

1953 年，Roth 利用圆法的变种证明了整数集的正密度子集一定含有非平凡的 3 项等差数列。近些年，二次方程在稠密集合上的解的存在性问题引起了学者们的大量研究。本报告将简介这方面的若干进展。

Romanoff's theorem for polynomials over finite fields revisited

周海燕

南京师范大学

Let g be a given polynomial of positive degree over a finite field. Shparlinski and Weingartner proved that the proportion of monic polynomials of degree n which can be represented by $h+g^k$ has the order of magnitude $1/\deg g$, where h is chosen from the set of irreducible monic polynomials of degree n and $k \in \mathbb{N}$. In this talk, we show that the proportion of monic polynomials of degree n which can be written as $l+g^p$ where l is the product of two monic irreducible polynomials with $\deg l = n$ and p is a prime number, still has the order of magnitude $1/\deg g$.

Perfect and almost perfect linear Lee codes

周悦

国防科技大学

Given a positive integer r, an abelian group G and a subset $T = \{a_1, a_2, \cdots, a_n\} \subseteq G \setminus \{e\}$, if

- all elements in the multiset

$$\Psi := \left\{ * \; a_1^{\pm j_1} \cdots a_n^{\pm j_n} : 0 \leq \sum_{k=1}^{n} j_k \leq r, j_k \in \mathbb{Z}_{\geq 0} \; * \right\}$$

  are distinct, and

- $G = \Psi$

then we call the Cayley graph $\Gamma(G, S)$ an *Abelian-Cayley-Moore graph*, where $S := T \cup T^{(-1)}$. Under this condition, the size of G (i.e. Ψ) is

$$\sum_{i=0}^{\min\{n,r\}} 2^i \binom{n}{i}\binom{r}{i}. \tag{1}$$

It is a bit surprising that the existence of an Abelian-Cayley Moore graph is equivalent to a perfect linear Lee code of radius r in $\mathbb{Z}^n$, that is a lattice tiling of $\mathbb{Z}^n$ by the translations of an ℓ_1-metric sphere of radius r. More than 50 years ago, Golomb and Welch [1] conjectured that there is no perfect Lee codes C for $r \geq 2$ and $n \geq 3$. Recently, Leung and the speaker [2] proved that if C is linear, then Golomb-Welch conjecture is true for $r = 2$ and $n \geq 3$.

In this talk, we consider the classification of linear Lee codes with the second best possibility, that is the density of the lattice packing of $\mathbb{Z}^n$ by Lee spheres $S(n, r)$ equals $\frac{|S(n,r)|}{|S(n,r)|+1}$. By checking the corresponding abelian Cayley graphs, an almost perfect linear Lee code is equivalent to the case with $G = \Psi \cup \{f\}$ where f is the unique element of order 2 in G. By a similar method used in [2], a classification result for $r = 2$ can also be obtained, but the computation is much more involved.

In the end of my talk, I will give a natural generalization of the bound (1) for elementary binary groups and show that the graph meeting this bound is equivalent to a binary perfect code with respect to the Hamming metric.

参考文献

[1] S. W. Golomb and L. R. Welch. Perfect codes in the Lee metric and the packing of polyominoes. *SIAM Journal on Applied Mathematics*, 18(2):302–317, 1970.

[2] K. H. Leung and Y. Zhou. No lattice tiling of $\mathbb{Z}^n$ by Lee sphere of radius 2. *Journal of Combinatorial Theory, Series A*, 171:105157, 2020.

Primitive matrix sets and synchronizing automata

祝隐峰

上海交通大学

A set $\mathcal{M}$ of $n \times n$ Boolean matrices is called primitive if there exists a positive product of matrices of $\mathcal{M}$ and the length of the shortest positive product is called primitive index. An automaton is a set $\mathcal{M}$ of $n \times n$ Boolean matrices such that there exists a unique 1 in each row of each matrix in $\mathcal{M}$. An automaton $\mathcal{M}$ is called synchronizing if there exists a product of matrices of $\mathcal{M}$ which has one positive column and the length of the shortest such product is called synchronizing index. Primitive property and synchronizing property have close connections. In this talk, we will introduce some variations of primitive/synchronizing property and primitive/synchronizing index. We will report some results about how to determine these properties and how to estimate these indices.

This is joint work with Yaokun Wu.
