2023天津图论组合青年研讨会

会议日程

8月11日	全天	会议注册，地点：汇高花园酒店一楼大堂
17:30-19:00	晚餐 汇高花园酒店 汇百厅	

8月12日	上午	数学科学学院 第二报告厅
8:45-9:00	开幕式	
主持人	洪振木	
9:00-9:40	孔将旭	Surviving rate of graphs for the Firefighter Problem
9:40-10:00	茶 歇	
主持人	蔡庆琼	
10:00-10:40	熊彦禛	A solution and some applications for a collapsibility problem of grid point set
10:40-11:20	祝隐峰	Primitivity and Hurwitz Primitivity of Nonnegative Matrix Tuples: A Unified Approach
11:30-12:30	午餐 汇高花园酒店 汇高C厅	
8月12日	下午	数学科学学院 第二报告厅
主持人	胡杰	
14:00-14:40	常渝林	Powers of tight Hamiltonian cycles in randomly perturbed hypergraphs
14:40-15:20	杨帆	On the size of (K_t, K_{1,k})-co-critical graphs
15:20-15:50	茶 歇	
主持人	王周宁馨	
15:50-16:30	房宜宾	Characterizing forbidden pairs for relative length of longest paths and cycles
16:30-17:10	胡杰	Spanning structures in incompatibility systems
18:00-20:00	晚餐 徽商故里	

8月13日	上午	数学科学学院 第二报告厅
主持人	房宜宾	
9:00-9:40	吴蓉	Graphs with girth 9 and without longer odd holes are 3-colorable
9:40-10:20	祖春蕾	Judicious bipartitions of graphs and directed graphs
10:20-11:00	自由讨论	
11:30-13:30	午餐 汇高花园酒店 汇百厅	
8月13日	下午	自由讨论

Powers of tight Hamiltonian cycles in randomly perturbed hypergraphs

Yulin Chang

Shandong University

Abstract

For integers $k \geq 3$ and $r \geq 2$, we show that for every $\alpha > 0$, there exists $\varepsilon > 0$ such that the union of k-uniform hypergraph on n vertices with minimum codegree at least αn and a binomial random k-uniform hypergraph $G^{(k)}(n,p)$ with $p \geq n^{-\binom{k+r-2}{k-1}^{-1}-\varepsilon}$ on the same vertex set contains the r^{th} power of a tight Hamilton cycle with high probability. Moreover, a construction shows that one cannot take $\varepsilon > C\alpha$, where $C = C(k,r)$ is a constant. Thus the bound on p is optimal up to the value of ε and this answers a question of Bedenknecht, Han, Kohayakawa, and Mota. This is a joint work with Jie Han and Lubos Thoma.

Characterizing forbidden pairs for relative length of longest paths and cycles

Yibin Fang

Nankai University

Abstract

The relative length of a graph is the difference of the order of a longest path and a longest cycle. In this talk, we present a complete characterization of the forbidden pairs $\mathcal{H}$ such that every 2-connected $\mathcal{H}$-free graph G satisfies $p(G) - c(G) \leq 1$, where $p(G)$ and $c(G)$ denote the order of a longest path and a longest cycle of G, respectively. We also investigate the forbidden pairs needed to guarantee a 2-connected $\mathcal{H}$-free graph has constant bounded relative length. These results extend the work of Faudree and Gould (1997) on Hamilton cycles.

Spanning structures in incompatibility systems

Jie Hu

Nankai University

Abstract

One of the most fundamental research topics in extremal graph theory is to determine sufficient conditions forcing spanning structures. Recently, there has been increasing interest in the study of robustness of graph properties, aiming to strengthen classical results in extremal and probabilistic combinatorics. The notion of incompatibility system, first proposed by Krivelevich, Lee and Sudakov, is a measure of robustness. In this talk, we will introduce robust version of some classical results about the existence of spanning structures, with respect to incompatibility systems.

Surviving rate of graphs for the firefighter problem

Jiangxu Kong

School of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 311121

Abstract

Let G be a connected graph with $n \geq 2$ vertices. Let $k \geq 1$ be an integer. Suppose that a fire breaks out at a vertex v of G. A firefighter starts to protect vertices. At each step, the firefighter protects k vertices not yet on fire. At the end of each step, the fire spreads to all the unprotected vertices that have a neighbour on fire. The process ends when the fire can no longer spread. The problem can be considered as a model for the propagation of viruses, rumors or epidemics. Let $sn_k(v)$ denote the maximum number of vertices in G that the firefighter can save when a fire breaks out at vertex v. The k-surviving rate $\rho_k(G)$ of G is defined to be $\frac{1}{n^2}\sum_{v\in V(G)} sn_k(v)$, which is the average proportion of saved vertices. In this talk, I will first give a chief survey on this topic, then prove that every oriented IC-planar graph $\overrightarrow{G}$ has $\rho_2(\overrightarrow{G}) > \frac{1}{104}$, which is an improvement for the result of oriented planar graphs.

Graphs with girth 9 and without longer odd holes are 3-colorable

Rong Wu

Shanghai Jiao Tong University

Abstract

For a number $l \geq 2$, let $\mathcal{G}_l$ denote the family of graphs which have girth $2l+1$ and have no odd hole with length greater than $2l+1$. Wu, Xu and Xu conjectured that every graph in $\bigcup_{l\geq 2} \mathcal{G}_l$ is 3-colorable. Chudnovsky et al., Wu et al., and Chen showed that every graph in $\mathcal{G}_2$, $\mathcal{G}_3$ and $\bigcup_{l\geq 5} \mathcal{G}_l$ is 3-colorable respectively. In this paper, we prove that every graph in $\mathcal{G}_4$ is 3-colorable. This confirms Wu, Xu and Xu's conjecture.

A solution and some applications for a collapsibility problem of grid point set

Yanzhen Xiong

University of Science and Technology of China

Abstract

For each positive integer n, let $[n]$ denote the set $\{1, \ldots, n\}$. For positive integers $a_1, \ldots, a_n$, the n-grid Gr of type $(a_1, \ldots, a_n)$ is the set $[a_1] \times \cdots \times [a_n]$. For each $I = \{i_1, \ldots, i_k\}$, let p_I stand for the projection map defined on Gr sending $(t_1, \ldots, t_n)$ to $(t_{i_1}, \ldots, t_{i_k})$. A subset of Gr is called a grid point set. A k-slice of Gr is a grid point set of the form $\mathrm{p}_I^{-1}(\mathrm{p}_I(x))$, where $x \in \mathrm{Gr}$ and $I \in \binom{[n]}{n-k}$.

Here is the problem: What is the maximum size of a grid point set $X \subseteq [a_1] \times \cdots \times [a_n]$ if the elements of X can be enumerated as $x_1, \ldots, x_m$ such that there exists a k-slice Y satisfying $Y \cap \{x_1, \ldots, x_i\} = \{x_i\}$ for each $i \in [m]$.

In this talk, I will introduce one of our solutions to this problem. In this method, we determine the rank of rainbow inclusion matrix, which generalizes the results of Richard Wilson and Peter Frankl. After that, I will talk about some applications related to hyperplane arrangement and compatible partition system.

On the size of $(K_t, K_{1,k})$-co-critical graphs

Fan Yang
Shandong University

Abstract

Given graphs G, H_1, H_2, we write $G \to (H_1, H_2)$ if every {red, blue}-coloring of the edges of G contains a red copy of H_1 or a blue copy of H_2. A non-complete graph G is (H_1, H_2)-*co-critical* if $G \nrightarrow (H_1, H_2)$, but $G + e \to (H_1, H_2)$ for every edge e in $\overline{G}$. Motivated by a conjecture of Hanson and Toft from 1987, we study the minimum number of edges over all $(K_t, K_{1,k})$-co-critical graphs on n vertices. We prove that for all $t \geq 3$ and $k \geq 3$, there exists a constant $\ell(t, k)$ such that, for all $n \geq (t-1)k + 1$, if G is a $(K_t, K_{1,k})$-co-critical graph on n vertices, then

$$e(G) \geq \left(2t - 4 + \frac{k-1}{2}\right) n - \ell(t, k).$$

Furthermore, this linear bound is asymptotically best possible when $t \in \{3, 4, 5\}$ and all $k \geq 3$ and $n \geq (2t-2)k + 1$. It seems non-trivial to construct extremal $(K_t, K_{1,k})$-co-critical graphs for $t \geq 6$. We also obtain the sharp bound for the size of $(K_3, K_{1,3})$-co-critical graphs on $n \geq 13$ vertices by showing that all such graphs have at least $3n - 4$ edges. Joint work with Hunter Davenport and Zi-Xia Song.

Primitivity and hurwitz primitivity of nonnegative matrix tuples: a unified approach

Yinfeng Zhu
Ural Federal University

Abstract

For an m-tuple of nonnegative $n \times n$ matrices $(A_1, \ldots, A_m)$, primitivity/Hurwitz primitivity means the existence of a positive product/Hurwitz product respectively (all products are with repetitions permitted). The Hurwitz product with a Parikh vector $\alpha = (\alpha_1, \ldots, \alpha_m) \in \mathbb{Z}^m_{\geq 0}$ is the sum of all products with α_i multipliers A_i, $i = 1, \ldots, m$. Ergodicity/Hurwitz ergodicity means the existence of the corresponding product with a positive row.

We give a unified proof for the Protasov-Vonyov characterization (2012) of primitive tuples of matrices without zero rows and columns and for the Protasov

characterization (2013) of Hurwitz primitive tuples of matrices without zero rows. By establishing a connection with synchronizing automata, we, under the aforementioned conditions, find an $O(n^2m)$-time algorithm to decide primitivity and an $O(n^3m^2)$-time algorithm to construct a Hurwitz primitive vector α of weight $\sum_{i=1}^{m} \alpha_i = O(n^3)$. We also report results on ergodic and Hurwitz ergodic matrix tuples.

Judicious bipartitions of graphs and directed graphs

Chunlei Zu

Shanghai Jiao Tong University

Abstract

Let D be a directed graph with n vertices, m arcs, and minimum outdegree at least d. Lee, Loh and Sudakov conjectured that the directed graph D admits a bipartition $V(D) = V_1 \cup V_2$ such that $\min\{e(V_1, V_2), e(V_2, V_1)\} \geq \big(\frac{d-1}{2(2d-1)} - o(1)\big)m$, where $e(V_i, V_j)$ denotes the number of arcs in D from V_i to V_j for $i \neq j \in \{1, 2\}$. For each $v \in V$, let $d^+(v)$ and $d^-(v)$ be the number of v's out-neighbors and in-neighbors, respectively. Let $s(v) = d^+(v) - d^-(v)$ and $\rho = \max_{u,v \in V} |s(u) - s(v)|$. We prove that D admits a bipartition $V(D) = V_1 \cup V_2$ with $\min\{e(V_1, V_2), e(V_2, V_1)\} > \frac{|V_1||V_2|}{n^2} m - \frac{\rho}{4} - \frac{1}{2}$. Moreover, if $\big||V_1| - |V_2|\big| \leq 1$, then D has a bisection $V = V_1 \cup V_2$ such that $\min\{e(V_1, V_2), e(V_2, V_1)\} > \frac{m-\rho-2}{4}$. The method can be used to some other problems. This is joint work with Qinghou Zeng.