
提交时间: October 29, 2019

Some notations

- p : prime;
- ϕ : Euler's totient function

1. Show that $\phi(mn) = \phi(m)\phi(n)\frac{d}{\phi(d)}$ for all m, n .

2. Show that if $d|n$ then $\phi(d)|\phi(n)$.

3 (Optional). Show that $n|\phi(a^n - 1)$ for all $a, n > 1$.

4. Show that for each integer m there are only finitely many n such that $\phi(n) = m$.

5. Find the last two digits of $\underbrace{3^{3^{\dots^3}}}_{2019}$.

6. One of Euler's "Sum of powers conjecture" was disproved in 1966 by finding the equality

$$133^5 + 110^5 + 84^5 + 27^5 = n^5$$

for some n . Can you find such n ?

7. Show that Jordan's totient function $J_k(n) = n^k \prod_{p|n} (1 - p^{-k})$.

8. Show that the inverse of ϕ is given by

$$\phi^{-1}(n) = \sum_{d|n} d\mu(d) = \prod_{p|n} (1 - p).$$

