Digraph homorphisms with the path-lifting property ⋆

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Abstract. For a digraph P, we say that a digraph homomorphism ϕ from G to H has the P-lifting property if for any homomorphism τ from P to H there exists a digraph homomorphism γ such that $\tau = \phi \circ \gamma$.
A digraph homomorphism is called path-liftable if it has the P-lifting property for all path digraphs P. If H is a strongly connected digraph but not a cycle, we show that it is NP-complete to decide whether or not a homomorphism from a digraph to H is path-liftable. For two strongly connected digraphs G and H of the same spectral radius, we give a cubic time algorithm to test whether or not a homomorphism from G to H is path-liftable. We show that a digraph homomorphism ϕ from an n-vertex digraph G to another digraph is path-liftable if it has the P-lifting property for all path digraphs P of length at most $2^n - 1$.

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